



TONBRIDGE SCHOOL

Scholarship Examination 2007

MATHEMATICS I

Tuesday 1st May 2007

9.00 a.m.

Time allowed: 1 hour 30 minutes

*Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.*

*All answers must be supported by adequate explanation.
Calculators may be used in any question.*

1. Solve the simultaneous equations:

$$2x + 5y = 23$$

$$5x - 3y = 11$$

Use your answers to find all the solutions of the equations:

$$2x^2 + 5\sqrt{y} = 23$$

$$5x^2 - 3\sqrt{y} = 11$$

[8 marks]

2. In a number such as 3459 we say that 9 (on the extreme right) is the *units digit*. In a number such as 18^2 , the units digit is 4 because $18^2=324$.

- (a) Find the units digits of the following numbers:

$$2007, 2007^2, 2007^3, 2007^4, 2007^5.$$

- (b) What do you notice about the pattern of units digits in (a)?

- (c) Use your answer to (b) to predict the units digits of (i) 2007^{10} , (ii) 2007^{21} , (iii) 2007^{43} . [8]

3. A plank of wood of width W rests horizontally on two supports, distance L apart. When a person of mass M sits in the middle of the plank, it sags by an amount S given (in appropriate units) by the formula $S = \frac{ML^3}{8000W}$.

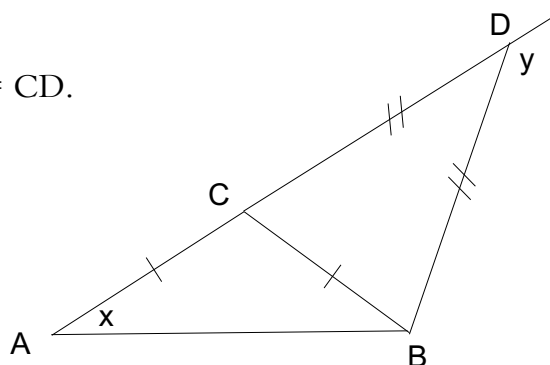
- (a) Given that $M = 62$, $L = 2.6$, $W = 0.7$, find S .

- (b) Given that $M = 70$, $L = 3.4$, $S = 0.4$, find W .

- (c) Given that $S = 1.1$, $W = 0.5$, $L = 4.0$, find M .

- (d) Given that $S = 0.8$, $W = 0.3$, $M = 65$, find L . [8]

4. In the diagram, the marked lengths are equal so that $AC = BC$ and $BD = CD$.



- (a) If $x = 15^\circ$, find y .

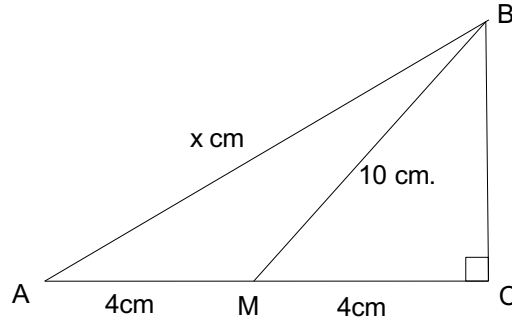
- (b) In general, what is the connection between x and y ?

- (c) If A and B are fixed and the position of C is moved so that x gets closer and closer to 45° , what happens to the position of D? [8]

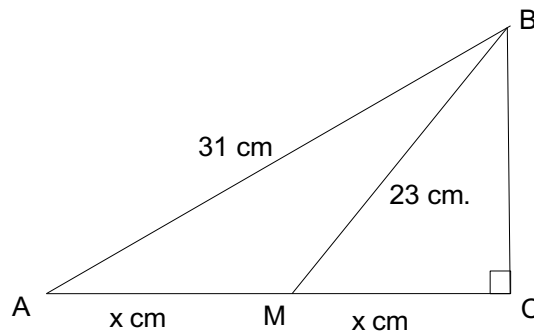
5. A field contains sheep and hens. The mean (average) number of legs of all the sheep and hens is 3.2. If there was one more sheep and one fewer hens, the mean number of legs would be 3.3. What is the total number of sheep and hens?

[8]

6. (a) In the diagram, M is the midpoint of AC . $AM = MC = 4$ cm, $BM = 10$ cm and angle ACB is 90° . Find the value of x .



- (b) In a similar diagram, $AB = 31$ cm, $BM = 23$ cm, $AM = MC = x$ cm and angle ACB is 90° . Use Pythagoras's theorem in triangles ABC and MBC to find two expressions for BC^2 and hence find the value of x .

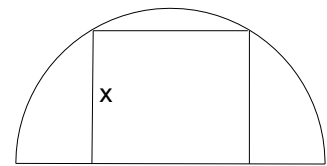


[15]

7. The diagram shows a rectangle with height x cm inscribed in a semicircle of radius 8 cm.

The area of the rectangle, y cm², is given by the formula

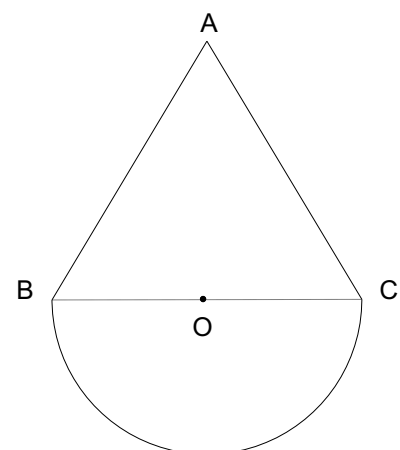
$$y = 2x\sqrt{64 - x^2}.$$



- (a) When $x = 4$, show that $y = 55.43$.
- (b) By referring to the diagram, not the formula, explain why $y = 0$ both when $x = 0$ and when $x = 8$.
- (c) Find the values of y for $x = 1, 2, 3, 5, 6, 7, 8$.
- (d) With a scale of 2 cm = 10 units on the y -axis and 1 cm = 1 unit on the x -axis, plot a graph of y against x .
- (e) Work out a few more values of y to try to find the highest value of y on the graph. What is the connection between the highest value of y and the radius of the circle?

[15]

8. The shape shown consists of an isosceles triangle ABC in which $AB = AC$ with base BOC , which is the diameter of a semicircle, centre O , of radius 5 cm. The height, AO , of the triangle is x cm.

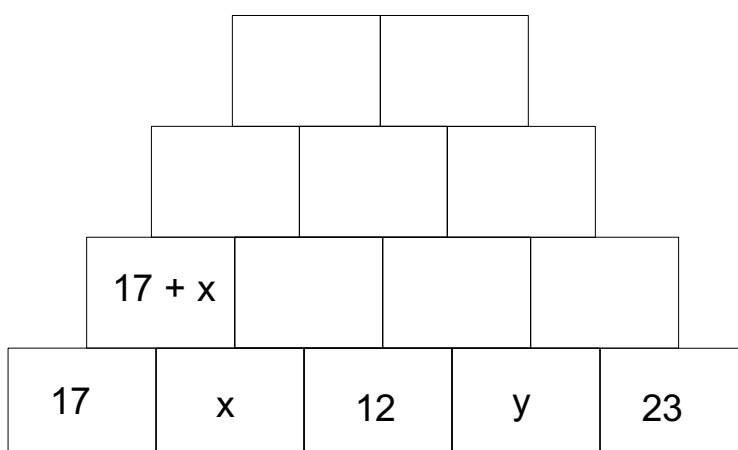


- (a) Find x if the area of the shape is the same as that of a circle of radius 5 cm.
- (b) Find x if the perimeter of the shape is the same as that of a circle of radius 5 cm.

[15]

9. In the diagram below, each box contains a number which is the sum of the numbers in the two boxes on which it rests.

For example, the box above those containing 17 and x contains $17 + x$.



- (a) Copy and complete the diagram with appropriate expressions involving x and y in each box.
- (b) If, in the top row, the left-hand box contains 79 and the right-hand box 73, find the values of x and y .
- (c) How would your answer to (b) change if, in the top row, the left-hand box contains 73 and the right-hand box 79?

[15]

